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Non-equilibrium relaxation at a tricritical point

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Abstract. We study the purely relaxational tricritical dynamics of a non-conserved order parameter (model A) in the upper critical dimension $d_c = 3$ and in $3 - \epsilon$ dimensions. We are especially interested in the relaxation, starting from a macroscopically prepared initial state with a small correlation length. Using the methods of renormalized field theory we obtain the scaling behaviour of the correlation and response functions and study the nonlinear relaxation of the order parameter $M(t)$. In three dimensions $M(t)$ displays a crossover from the purely logarithmic short-time behaviour $M(t) \sim (\ln(t/t_0))^{-a}$ to a $t^{-1/4}$ power law with logarithmic corrections.

For dimensions $d < 3$ we obtain the exponents which govern the tricritical relaxation at lowest non-trivial order in $\epsilon = 3 - d$. The dynamic scaling exponent z is calculated at second order in ϵ.

1. Introduction

The properties of thermodynamic systems near tricritical points have been thoroughly studied over recent decades [1]. A considerable part of the progress achieved in this field is due to the application of renormalization group methods which permit a deeper understanding of scaling behaviour and constitute a tool for the systematic calculation of (tri-)critical exponents, logarithmic corrections and scaling functions.

Siggia and Nelson [2] have analysed the tricritical dynamics of ^{3}He–^{4}He mixtures and antiferromagnetic systems near four dimensions and obtained scaling functions which describe the crossover from tricritical to λ-line behaviour. However, since in this approach the tricritical point corresponds to a Gaussian fixed point the logarithmic corrections to the mean-field-like behaviour in three dimensions cannot be found.

In this paper, we study the tricritical purely relaxational dynamics of a non-conserved order parameter (model A) in the upper critical dimension $d_c = 3$ and in $3 - \epsilon$ dimensions. Our special interest is focused on the relaxation, starting from a non-equilibrium initial state with short-range correlations. Only a few years ago it was shown that even the (macroscopically) early stage of the *critical* relaxation displays universal scaling behaviour [3, 4]. We extend these studies to the case of tricritical points and find that the order parameter $M(t)$ ('magnetization') in three dimensions is given by

$$M(t) = M_0(\ln(t/t_0))^{-a} F_M\left(\left(\frac{t}{\ln(t/t_0)}\right)^{1/4} (\ln(t/t_0))^{-a} M_0\right) \tag{1}$$

where

$$F_M(x) \sim \begin{cases} 1 & \text{for } x = 0 \\ 1/x & \text{for } x \to \infty \end{cases} \tag{2}$$

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and a is an universal exponent. For a one-component order parameter $a = 3/(40\pi)$. Hence the order parameter relaxation shows logarithmic behaviour at short times (but large with respect to microscopic relaxation) $t_0 \ll t \ll M_0^{-4}$ and a crossover to $t^{-1/4}$ behaviour with logarithmic corrections.

In addition we obtain the dynamic scaling exponent z for $d < 3$ at second order in $\epsilon = 3 - d$.

2. The model

We consider the purely relaxational order parameter dynamics near a tricritical point without any coupling to a conserved density. In this case the only slow mesoscopic variable is the order parameter field itself. The analysis starts from the Langevin equation

$$\partial_t s(x,t) = -\lambda \frac{\delta \mathcal{H}[s]}{\delta s(x,t)} + \zeta(x,t) \tag{3}$$

where $s = (s_1, \ldots, s_n)$ denotes the n-component order parameter field with $O(n)$ symmetry, and $\zeta = (\zeta_1, \ldots, \zeta_n)$ is a Gaussian white noise with zero mean and the correlations

$$\langle \zeta_\alpha(x,t)\zeta_\beta(x',t')\rangle = 2\lambda\delta_{\alpha\beta}\delta(x-x')\delta(t-t'). \tag{4}$$

The Hamiltonian $\mathcal{H}$ is given by

$$\mathcal{H}[s] = \int \mathrm{d}^d x \left[\frac{r}{2}s^2 + \frac{1}{2}(\nabla s)^2 + \frac{g}{4!}(s^2)^2 + \frac{f}{6!}(s^2)^3\right]. \tag{5}$$

In mean-field theory the tricritical point corresponds to $r = g = 0$. An equivalent formulation for the tricritical dynamics is given by the stochastic functional [4–7]

$$\mathcal{J}[\tilde{s}, s] = \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x \tilde{s}\left[\partial_t s + \lambda(r - \Delta)s + \frac{\lambda g}{3!}(s^2)s + \frac{\lambda f}{5!}(s^2)^2 s - \lambda\tilde{s}\right] \tag{6}$$

where $\tilde{s}$ is a Martin–Siggia–Rose response field [8]. The weight $\exp(-\mathcal{J})$ integrated over the response field $\tilde{s}$ may be interpreted as a path probability distribution of the stochastic process $\{s(x,t)\}$ starting from an initial configuration $\{s_0(x)\}$.

Since we wish to study the influence of initial conditions corresponding to a macroscopically prepared initial state with short-range correlations we also average over $s_0(x) = s(x,0)$ with a probability distribution $\exp(-\mathcal{H}^{(i)}[s_0])$, where [3, 4]

$$\mathcal{H}^{(i)}[s_0] = \int \mathrm{d}^d x \tau_0/2(s_0(x) - M_0)^2. \tag{7}$$

By naive dimensional analysis, $\tau_0 \sim \mu^2$ (where μ is an external momentum scale), i.e. τ_0^{-1} corresponds to an irrelevant parameter, and the distribution $\exp(-\mathcal{H}^{(i)})$ is equivalent (up to corrections to scaling) to the sharp initial condition $s_0(x) = M_0$.

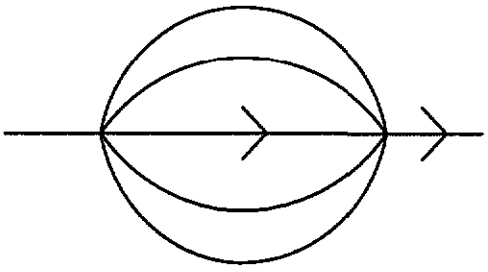

Figure 1. Contribution to $\mathring{\Gamma}_{1,1}$ at lowest non-trivial order.

3. Renormalization group analysis

Given $\mathcal{J}$ and $\mathcal{H}^{(i)}$, the program for studying the tricritical relaxation follows the lines taken in [3, 4, 9, 10]. Upon expanding $\exp(-\mathcal{J})$ in powers of f and g we obtain the perturbation series of the Green functions

$$G^{\tilde{L}}_{\tilde{N},N}(\{x,t\}) = \langle [\tilde{s}_0]^{\tilde{L}} [\tilde{s}]^{\tilde{N}} [s]^N \rangle \tag{8}$$

where $\tilde{L}$ response fields are fixed to $t = 0$. While the free propagator

$$G_q(t-t') = \theta(t-t') \exp(-\lambda(r+q^2)(t-t')) \tag{9}$$

remains unchanged by the initial conditions, the correlator is given by

$$C_q(t,t') = C_q^{(\mathrm{eq})}(t-t') + C_q^{(\mathrm{i})}(t,t') \tag{10}$$

where

$$C_q^{(\mathrm{eq})}(t-t') = \frac{1}{r+q^2} \exp(-\lambda(r+q^2)|t-t'|) \tag{11}$$

is the equilibrium correlator, and

$$C_q^{(\mathrm{i})}(t,t') = \left(\tau_0^{-1} - \frac{1}{r+q^2}\right) \exp(-\lambda(r+q^2)(t+t')). \tag{12}$$

From equations (9)–(12) we see that we can set $s_0 = \tilde{s}_0/\tau_0$, $\dot{s}_0 = \lambda \tilde{s}_0$ in the correlation functions. Thus only the Green functions (8) have to be studied.

The renormalized field theory follows the usual lines [11]. We use dimensional regularization to render the ultraviolet divergent integrals finite and absorb the remaining poles at $\epsilon = 0$ into reparametrizations of the coupling constants and fields:

$$\begin{aligned}
&s \to \mathring{s} = Z_s^{1/2} s \qquad \tilde{s} \to \mathring{\tilde{s}} = Z_{\tilde{s}}^{1/2} \tilde{s} \\
&\lambda \to \mathring{\lambda} = (Z_s/Z_{\tilde{s}})^{1/2} \lambda \qquad f \to \mathring{f} = B_\epsilon^{-2} Z_s^{-3} Z_v v \mu^{2\epsilon} \\
&g \to \mathring{g} = B_\epsilon^{-1} Z_{\tilde{s}}^{-2} Z_u u \mu^\epsilon \qquad r \to \mathring{r} = Z_s^{-1}(Z_r r + W u^2).
\end{aligned} \tag{13}$$

The factor $B_\epsilon = \Gamma((1-\epsilon)/2)/(4\pi)^{d/2}$ has been introduced for convenience.

The Z-factors defined above suffice to render equilibrium Green functions finite. However, the non-equilibrium initial conditions break the translational invariance with respect to time and require an additional renormalization of the field $\tilde{s}_0$:

$$\tilde{s}_0 \to \mathring{\tilde{s}}_0 = (Z_0 Z_{\tilde{s}})^{1/2} \tilde{s}_0. \tag{14}$$

At lowest non-trivial order the Z-factors are

$$Z_s = 1 - \frac{(n+2)(n+4)}{2700\epsilon} v^2 + O(v^3) \tag{15}$$

$$Z_r = 1 + \frac{(n+2)(n+4)}{180\epsilon} v^2 + O(v^3) \tag{16}$$

$$Z_u = 1 + \frac{4(n+4)}{15\epsilon} v + O(v^2) \tag{17}$$

$$Z_v = 1 + \frac{2(3n+22)}{15\epsilon} v + O(v^2) \tag{18}$$

$$W = \frac{n+2}{9\epsilon} + O(v). \tag{19}$$

These renormalization factors are already known from the work of Lawrie and Sarbach [1]. The renormalization constant $Z_{\tilde{s}}$ is obtained by requiring that the single-particle irreducible *equilibrium* vertex function $\Gamma_{1,1} = (Z_s Z_{\tilde{s}})^{1/2} \mathring{\Gamma}_{1,1}$ be finite in $d = 3$. The evaluation of the Feynman graph (figure 1) yields

$$Z_s Z_{\tilde{s}} = 1 - \frac{(n+2)(n+4)}{450\epsilon} I v^2 + O(v^3) \tag{20}$$

where

$$I = \int_0^\infty \frac{dx}{x^3} (\mathrm{erf}(x))^5 = 0.648\,555. \tag{21}$$

(Here erf is the error function [12].)

In order to obtain well-defined renormalized *non-equilibrium* Green functions with insertions of the field $\tilde{s}_0$ we have to render the response function

$$G^1_{0,1}(x,t) = (Z_s Z_{\tilde{s}} Z_0)^{-1/2} \mathring{G}^1_{0,1}(x,t) \tag{22}$$

finite. At first order in $\mathring{f}$ we find (for $\tau_0^{-1} = M_0 = 0$)

$$\int d^d x \mathring{G}^1_{0,1}(x,t;\mathring{r}=0,\mathring{g}=0) = 1 - \frac{(n+2)(n+4)}{5!} \frac{\mathring{f}}{(4\pi)^d} \frac{2}{(1-\epsilon)^2\epsilon} (2\mathring{\lambda}t)^\epsilon + O(\mathring{f}^2). \tag{23}$$

The corresponding Feynman graph is shown in figure 2. Equation (23) yields for the Z-factor

$$Z_0 = 1 - \frac{(n+2)(n+4)}{30\pi\epsilon} v + O(v^2). \tag{24}$$

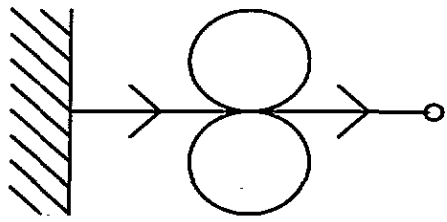

Figure 2. Contribution to $\mathring{G}^{1}_{0,1}$ at first order in the s^6 coupling. The hatched area represents $t = 0$.

We can now exploit the μ independence of the bare Green functions

$$\mathring{G}^{\tilde{L}}_{\tilde{N},N} = (Z_0 Z_{\tilde{s}})^{\tilde{L}/2} Z_{\tilde{s}}^{\tilde{N}/2} Z_s^{N/2} G^{\tilde{L}}_{\tilde{N},N} \tag{25}$$

to obtain the renormalization group equations (RGEs)

$$[\mu\partial_\mu + (r\kappa_r + u^2\kappa_{ur})\partial_r + \kappa_u u\partial_u + \zeta\lambda\partial_\lambda + \beta\partial_v + (\gamma_0 + \gamma_{\tilde{s}})\tilde{L}/2 + \gamma_{\tilde{s}}\tilde{N}/2 + \gamma_s N/2]G^{\tilde{L}}_{\tilde{N},N} = 0 \tag{26}$$

for the renormalized functions $G^{\tilde{L}}_{\tilde{N},N}$. The Wilson functions are defined as derivatives at fixed bare parameters:

$$\gamma_\alpha = \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\bigg|_0 \ln Z_\alpha \qquad (\alpha = 0, \tilde{s}, s, r, u, v) \qquad \kappa_r = \gamma_s - \gamma_r$$

$$\kappa_u = 2\gamma_s - \gamma_u - \epsilon \qquad \zeta = (\gamma_{\tilde{s}} - \gamma_s)/2$$

$$\beta = (-2\epsilon + 3\gamma_s - \gamma_v)v$$

and

$$\kappa_{ur} = 2(1 + v\partial_v)W^{(1)} \qquad \text{with } W = W^{(1)}/\epsilon + \mathrm{O}(1/\epsilon^2).$$

At lowest non-trivial order we have

$$\beta = v(-2\epsilon + \frac{3n+22}{15}4v + \mathrm{O}(v^2))$$

$$\gamma_s = \frac{(n+2)(n+4)}{15}\frac{v^2}{45} + \mathrm{O}(v^3)$$

$$\kappa_r = \frac{16(n+2)(n+4)}{15}\frac{v^2}{45} + \mathrm{O}(v^3)$$

$$\kappa_u = -\epsilon + \frac{n+4}{5}\frac{8v}{3} + O(v^2)$$

$$\kappa_{ur} = \frac{n+2}{3}\frac{2}{3} + \mathrm{O}(v)$$

$$\zeta = (3I - 1)\gamma_s + \mathrm{O}(v^3)$$

$$\gamma_{\tilde{s}} = (6I - 1)\gamma_s + \mathrm{O}(v^3)$$

$$\gamma_0 = \frac{(n+2)(n+4)}{15\pi}v + \mathrm{O}(v^2).$$

Below three dimensions the coupling coefficient v possesses the non-trivial fixed point

$$v_* = \frac{15}{2}\frac{\epsilon}{3n+22} + O(\epsilon^2) \tag{28}$$

and the solution of the RGE (26) in conjunction with dimensional analysis yields the scaling behaviour

$$G^{\tilde{L}}_{\tilde{N},N}(\{x,t\};\tau,u,v_*;\lambda,\mu) = l^{\tilde{L}(\eta_0+\tilde{\eta})/2+\tilde{N}\tilde{\eta}/2+N\eta/2} l^{(\tilde{L}+\tilde{N})(d+2)/2+N(d-2)/2}$$
$$\times G^{\tilde{L}}_{\tilde{N},N}(\{lx, l^z t\}; l^{-y_\tau}\tau, l^{-y_u}u, v_*; \lambda, \mu) \tag{29}$$

with the well-known [13, 14] tricritical indices

$$y_\tau = 2 - \kappa_r^* = 2 - \frac{4(n+2)(n+4)}{3(3n+22)^2}\epsilon^2 + O(\epsilon^3) \tag{30}$$

$$y_u = 1 - \kappa_u^* = 1 + \frac{6-n}{3n+22}\epsilon + O(\epsilon^2) \tag{31}$$

$$\eta = \gamma_s^* = \frac{(n+2)(n+4)}{(3n+22)^2}\frac{\epsilon^2}{12} + O(\epsilon^3). \tag{32}$$

The scaling field τ in equation (29) is defined as $\tau = r + Au^2$, where

$$A = \kappa_{\tau u}^*/(2y_u - y_\tau) = \frac{5}{6}\frac{(n+2)}{(6-n)}\frac{1}{v_*}(1 + O(v_*)). \tag{33}$$

The dynamic scaling exponent $z = 2 + \zeta^* = 2 + (\tilde{\eta} - \eta)/2$ and the initial slip exponent $\eta_0 = \gamma_0^*$ follow from equations (20) and (24), respectively:

$$z = 2 + c\eta + O(\epsilon^3) \qquad c = 3I - 1 = 0.945\,666 \tag{34}$$

$$\eta_0 = \frac{(n+2)(n+4)}{3n+22}\frac{\epsilon}{2\pi} + O(\epsilon^2). \tag{35}$$

The autocorrelation function $C(t) = \langle s(x,t)s(x,0)\rangle = \tau_0\langle\tilde{s}_0 s(x,0)\rangle$ at the tricritical point is given by

$$C(t) \sim t^{-(d/z)+\theta'} \qquad \text{with } \theta' = (2 - z - \eta - \eta_0/2)/z. \tag{36}$$

For a one-component order parameter we have $\theta' = -3\epsilon/(40\pi) + O(\epsilon^2)$. This prediction may be checked by computer simulations as in [15].

In three dimensions, $v_* = 0$, and the scaling behaviour is governed by mean-field exponents with logarithmic corrections. The flow equation

$$l\frac{d}{dl}\bar{v}(l) = b\bar{v}(l)^2 + O(\bar{v}(l)^3) \qquad b = 4\frac{3n+22}{15} \tag{37}$$

leads asymptotically to the solution

$$\bar{v}(l) \simeq 1/(b\ln(1/l)) \qquad \text{for } l \ll 1. \tag{38}$$

If we define scaling fields [1]

$$\tau = r + \frac{5}{6}\frac{(n+2)}{(6-n)}\frac{u^2}{v} \tag{39}$$

$$\sigma = u/\sqrt{v} \tag{40}$$

the RGEs yield the scaling of the correlation functions (up to non-universal factors)

$$G^{\tilde{L}}_{\tilde{N},N}(\{x,t\};\tau,\sigma,v;\lambda,\mu)$$

$$= l^{(N+5\tilde{N}+5\tilde{L})/2}\left[\frac{\bar{v}(l)}{v}\right]^{\tilde{L}a} G^{\tilde{L}}_{\tilde{N},N}\left(\{lx, l^2t\}; \frac{\tau}{l^2}, \frac{\sigma(\bar{v}(l)/v)^{-q_\sigma}}{l}, \bar{v}(l); \lambda, \mu\right) \tag{41}$$

where

$$q_\sigma = (6-n)/[2(3n+22)] \tag{42}$$

$$a = (n+2)(n+4)/[8\pi(3n+22)]. \tag{43}$$

The scaling behaviour of the autocorrelation function at the tricritical point follows from

$$C(t) = \tau_0 G^1_{0,1}(x=0,t;0,0,v;\lambda,\mu)$$

$$= \tau_0 l^3\left(\frac{\bar{v}(l)}{v}\right)^a G^1_{0,1}(x=0,l^2t;0,0,\bar{v}(l);\lambda,\mu). \tag{44}$$

We now exploit equation (44) choosing $l \sim (\lambda\mu^2 t)^{-1/2} \ll 1$. The last condition means that t has to be larger than a typical microscopic relaxation time t_0. In this way we get

$$C(t) \sim t^{-3/2}(\ln(t/t_0))^{-a}. \tag{45}$$

So far we have only considered the case of a vanishing initial order parameter $M_0 = 0$. In order to study the decay of a non-zero order parameter notice that with $\tau_0 s_0 = \tilde{s}_0$ [3,4] we have

$$M(t) = \langle s(x,t)\rangle = \int \mathcal{D}[\tilde{s},s]s(x,t)\exp(-\mathcal{J} - \mathcal{H}^{(i)}) \tag{46}$$

and

$$\mathcal{H}^{(i)} = -\int \mathrm{d}^d x M_0 \tilde{s}_0(x) + O(1/\tau_0). \tag{47}$$

Thus we can write $M(t)$ in the scaling limit as a power series in M_0

$$M(t) = \sum_{K=1}^{\infty}\frac{1}{K!}\int\ldots\int\prod_{k=1}^{K}\mathrm{d}^d x_k G^K_{0,1}(\{x,x_1,\ldots,x_k,t\};\tau,\sigma,v;\lambda,\mu)M_0^K$$

$$= \sum_{K=1}^{\infty}\frac{M_0^K}{K!}\left[\frac{\bar{v}(l)}{v}\right]^{Ka}(\sqrt{\mu l})^{l-K}$$

$$\times \hat{G}^K_{0,1}(\{q=\mathbf{0},\lambda\mu^2 l^2 t\};\tau/l^2,\sigma(\bar{v}(l)/v)^{-q_\sigma}/l,\bar{v}(l);1,1)$$

$$= M_0\left[\frac{\bar{v}(l)}{v}\right]^a F(\lambda\mu^2 l^2 t,\tau/l^2,\sigma(\bar{v}(l)/v)^{-q_\sigma}/l,\bar{v}(l),M_0(\bar{v}(l)/v)^a/\sqrt{\mu l}). \tag{48}$$

Choosing again $l \sim (\lambda\mu^2 t)^{-1/2} \ll 1$ we find at the tricritical point

$$M(t) = (\ln(t/t_0))^{-a} M_0 f(t^{1/4}(\ln(t/t_0))^{-a} M_0, \bar{v}((t/t_0)^{-1/2})). \tag{49}$$

Since for $v \to 0$ only tree diagrams contribute to the Green functions, the scaling function f (without the logarithmic corrections) can be calculated by mean-field theory. We have to solve the differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t} M(t) = -\frac{\lambda v}{5!} M(t) \tag{50}$$

with the result

$$M(t) = M_0(1 + \text{constant} \times v t M_0^4)^{-1/4}. \tag{51}$$

From this result we infer (up to non-universal factors)

$$M(t) = M_0(\ln(t/t_0))^{-a} F_M\left(\left(\frac{t}{\ln(t/t_0)}\right)^{1/4} (\ln(t/t_0))^{-a} M_0\right) \tag{52}$$

with

$$F_M(x) = 1/(1 + x^4)^{1/4}. \tag{53}$$

Hence the relaxation of the order parameter displays a crossover from the logarithmic power law

$$M(t) \sim M_0(\ln(t/t_0))^{-a} \qquad \text{for } t \ll M_0^{-4} \tag{54}$$

to the long-time behaviour

$$M(t) \sim \left(\frac{t}{\ln(t/t_0)}\right)^{-1/4} \qquad \text{for } t \gg M_0^{-4}. \tag{55}$$

Below three dimensions a calculation as in [3, 4] leads to the scaling form instead of equation (52)

$$M(t) = M_0 t^{\theta'} f_M(t^{\theta'} M_0) \tag{56}$$

with the exponent θ' given in equation (36).

4. Conclusions

In this paper, we have studied the tricritical dynamics of a thermodynamic system with a non-conserved order parameter, particularly the non-equilibrium relaxation, starting from an initial state with short-range correlations.

For dimensions $d < 3$ the decay of an initial order parameter $M_0 \neq 0$ can be described by a universal scaling law which exhibits a crossover between two power laws. We have calculated the new exponent θ', which characterizes the short-time behaviour, to first order in $\epsilon = 3 - d$ and obtained the dynamic scaling exponent z to second order in ϵ.

In three dimensions the order parameter relaxation displays a crossover from purely logarithmic behaviour at short times $t \ll M_0^{-4}$ to a power law with a logarithmic correction for $t \gg M_0^{-4}$. This result may be applicable to antiferromagnets as e.g. dysprosium aluminum garnet (DAG) in which the order parameter (staggered magnetization) couples directly to an applied magnetic field along the [111] direction [16]. The induced staggered magnetic field is necessary to prepare the initial value $M_0 \neq 0$ of the order parameter.

Finally, we wish to point out that our results (for $d = 2$ as well as for $d = 3$) can be verified by computer experiments.

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